**Analysis Report**

**Summary to the VaR of three Portfolios**

**Growth Portfolio:**

Higher Volatility: Growth Portfolio typically invests in high-growth sectors like technology and biotech, which often exhibits a larger price swing because of uncertain earnings and market sentiment.

Tail Risk: The portfolio reflected a consistently highest VaR estimates for all methods. Indicating that there are rapid expansions or contractions in earning expectations and these can lead to higher tail risk.

**Value Portfolio:**

Stability: Value portfolio shows more stability and less sensitivity to market hype because value stocks usually represent mature companies with more stable cash flows.

Lower Volatility: Value portfolio has the lowest VaR among the three portfolios whatever methods. Historically, value stocks tend to show less dramatic price fluctuations, thus resulting in lower VaR.

**Industrial Portfolio:**

Risk Level in the Middle: The Industrial Portfolio shows a VaR between Value Portfolio and Growth Portfolio.

Sector Exposure: Industrial Portfolios are often comprised with diversified sector ETFs and they exhibit moderate volatility. Their risk is influenced by broader economic cycles and global demand.

**Method Differences in VaR Estimates and Sensitivity**

***Parametric Normal:***

**KeyAssumptions*:*** Assuming returns are normally distributed, and volatility is constant over the period. Then uses the portfolio’s mean and standard deviation

**Characteristic of VaR estimates:** It often yields lower VaR if the actual return distribution has fat tails or skewness, because the normal assumption underestimates extreme losses.

**Sensitivity:**Highly sensitive to the normality assumption. If returns exhibit non‑normal behavior, this method will underestimate extreme losses.

***Historical Simulation:***

**Key Assumptions:** Historical returns are representative of future outcomes. And returns are independent, identically distributed and stationary.

**Characteristic of VaR estimates:** If historical data includes severe drawdowns (encountering extreme events), VaR will be higher than other methods; if it’s mostly calm, VaR will be lower.

**Sensitivity:** The method highly depends on the historical window; if the sample is unrepresentative with some extreme values(encountering extreme events), it may overestimate risk.

***Monte Carlo:***

**Key Assumptions:** The method simulates future returns based on an assumed distribution (here normal). And the simulated returns are i.i.d.

**Characteristic of VaR estimates:** VaR accuracy depends on the assumed distribution. Normal-based simulations can underestimate tails, while heavier-tailed assumptions (t-distribution simulation) may produce higher VaR.

**Sensitivity:** It is highly sensitive to the assumed distribution and independence of returns in simulations.

***GPD (Extreme Value Theory):***

**Key Assumptions:** Extreme losses follow a Generalized Pareto Distribution. And a proper threshold is chosen and there’s sufficient tail data.

**Characteristic of VaR estimates:** The model focuses exclusively on extreme losses beyond a threshold using Extreme Value Theory, thus often yielding higher VaR when extreme losses are significant.

**Sensitivity:** The method is very sensitive to the choice of threshold and the amount of tail data available, like a small sample can lead to unstable estimates.

***Filtered Historical Simulation (FHS):***

**Key Assumptions:** A volatility model can filter out time‑varying volatility and yield standardized i.i.d. returns. The historical distribution of these standardized returns should be representative.

**Characteristic of VaR estimates:** Filters returns using a volatility model to remove volatility clustering, then applies historical simulation on standardized returns. So, it can produce higher VaR if recent volatility is elevated.

**Sensitivity:** The model is dependent on the accuracy of the volatility model; if the model is mis specified, the standardized returns might not be i.i.d., thus affecting VaR.

***GARCH-Based:***

**Key Assumptions:** Returns follow a GARCH process with predictable conditional variance. And innovations follow a specified distribution (normal or t‑distributed).

**Characteristic of VaR estimates:** The model applies a parametric VaR formula (assuming normality for residuals here). It reflects current market volatility; when volatility is high, forecasted variance (and thus VaR) is higher.

**Sensitivity:** Sensitive to model specification and the assumption on residual distribution; captures volatility clustering but may misestimate risk if the model is incorrect.

***Cornish-Fisher:***

**Key Assumptions:** The return distribution is approximately normal but exhibits skewness and excess kurtosis. And moments (mean, variance, skewness, kurtosis) are reliably estimated.

**Characteristic of VaR estimates:** The VaR can be either estimated higher or lower than the normal method depending on the magnitude and sign of skewness and kurtosis. Negative skew and high kurtosis generally push VaR higher.

**Sensitivity:** The model is very sensitive to the accurate estimation of skewness and kurtosis; if these moments are estimated poorly, the VaR adjustment can be misleading.

***Bootstrapping:***

**Key Assumptions:** Historical returns are i.i.d. and representative of future risk. And the historical sample sufficiently captures the distribution, including tail events.

**Characteristic of VaR estimates:** For each bootstrap iteration, it can yield a different tail quantile, resulting in a range of VaR estimates. This variability is accentuated if the dataset contains significant outliers. This can push the estimated risk higher, whereas a calmer period in the historical data leads to lower estimates.

**Sensitivity:** It relies on the quality and representativeness of the historical data; if extreme events are rare or missing, VaR may be underestimated. If comparatively large proportion of extreme values, VaR may be overestrimated.

***KDE:***

**Key Assumptions:** The return distribution is stationary and can be estimated non‑parametrically. And the chosen kernel and bandwidth accurately capture the distribution’s shape.

**Characteristic of VaR estimates:** The method may differ from historical simulation depending on the chosen kernel and bandwidth and it can capture multi‑modality or skew more flexibly.

**Sensitivity:** It is sensitive to the bandwidth selection; poor choices can lead to either underestimated or overestimated risk.

***Parametric t-Distribution:***

**Key Assumptions:** The method assumes returns follow a Student’s t‑distribution with heavy tails. And the degrees of freedom parameter correctly reflect tail behavior.

**Characteristic of VaR estimates:** Typically, it will yield higher VaR than the normal method if the degrees of freedom are small (fat tails)

**Sensitivity:** The method is sensitive to the chosen degrees of freedom; it better captures extreme events but relies on accurate parameter estimation.

**Sensitivity Summary**

***Normality:*** Methods assuming normality (e.g., Parametric Normal, some Monte Carlo) are very sensitive to any departure from the normal distribution. If returns have fat tails or skewness, it will underestimate tail risk because they miss the frequency and magnitude of extreme losses.

***Fat Tails:*** Methods designed for heavy tails (t-distribution, GPD, Cornish-Fisher) are sensitive to the parameters governing tail behavior. Small changes in degrees of freedom or threshold selection can lead to large differences in VaR estimates.

***Volatility Clustering:*** GARCH-based and Filtered Historical Simulation capture changing volatility. Ignoring it (e.g., a static σ) can under- or overestimate VaR.

***Historical Data Reliance:*** Methods including historical Simulation, Bootstrapping, and KDE directly depend on past observations. If the dataset lacks extreme events, these methods may underestimate tail risk. if it includes outliers, risk may be overstated.

***Parametric vs. Non-parametric:*** Parametric approaches (e.g. GARCH) are highly sensitive to model specification and forecast accuracy; non-parametric methods (historical simulation, bootstrapping) rely on the historical sample itself. Both can be misled if the chosen assumption or historical window is unrepresentative.

**Statistical test results for assumptions**

Value Portfolio

**Normality (Shapiro-Wilk, Kolmogorov-Smirnov):**

Shapiro-Wilk: p-value < 0.05 → Rejects normality.

Kolmogorov-Smirnov: p-value < 0.05 → Indicates non-normal distribution.

Overall, the Value portfolio exhibits significant deviation from normality.

**Autocorrelation (Ljung-Box):**

Ljung-Box (10 lags): p-value < 0.05 → Indicates statistically significant autocorrelation.

This suggests that returns are not fully independent or white noise over the tested lags.

**Tail Behavior (Anderson-Darling):**

Anderson-Darling: Statistic = 14.1363, well above the 1% critical value (1.0890).

This indicates a strong deviation in the tails from normality, implying heavier or very complex tail behavior.

**Conclusion:**

Non-normal returns with evidence of autocorrelation and heavy tails indicate that methods assuming strict normality or i.i.d. returns may underestimate risk.

Growth Portfolio

**Normality (Shapiro-Wilk, Kolmogorov-Smirnov):**

• Shapiro-Wilk: p-value < 0.05 → Rejects normality.

• Kolmogorov-Smirnov: p-value < 0.05 → Indicates non-normal distribution.

Overall, the Growth portfolio shows significant deviation from normality.

**Autocorrelation (Ljung-Box):**

Ljung-Box (10 lags): p-value < 0.05 → Indicates statistically significant autocorrelation.

This suggests that returns are not entirely independent, violating the i.i.d. assumption.

**Tail Behavior (Anderson-Darling):**

Anderson-Darling: Statistic = 7.0118, well above all critical values.

This signifies heavy tail behavior and marked deviation from normality in extreme returns.

**Conclusion:**

The Growth portfolio’s non-normal returns, combined with autocorrelation and heavy tails, suggest that methods assuming normality may significantly understate extreme risk.

Industrial Portfolio

**Normality (Shapiro-Wilk, Kolmogorov-Smirnov):**

• Shapiro-Wilk: p-value < 0.05 → Rejects normality.

• Kolmogorov-Smirnov: p-value = 0.0053 < 0.05 → Indicates non-normal distribution.

Overall, the Industrial portfolio deviates significantly from normality.

**Autocorrelation (Ljung-Box):**

Ljung-Box (10 lags): p-value ≈ 0.000004 → Indicates strong autocorrelation.

This suggests that the returns exhibit substantial dependence and volatility clustering.

**Tail Behavior (Anderson-Darling):**

Anderson-Darling: Statistic = 6.8400, well above the 1% critical value.

This indicates a strong departure in tail behavior, with heavy tails present.

**Conclusion:**

The Industrial portfolio exhibits non-normal returns, significant autocorrelation, and heavy tails; thus, risk may be underestimated by models that assume normality and independence.

**Coherent Risk Measure Analysis**

**1. Monotonicity**

In the table displayed in Python, this axiom is split into two subcategories: Monotonicity (Value) and Monotonicity (Growth). Most methods pass both tests, indicating they correctly assign higher risk to portfolios with systematically worse returns. However, the Cornish-Fisher method fails for both portfolios. This is a known issue with this method, as it uses a Gram-Charlier expansion which may yield non-monotonic transformations of the underlying distribution, especially in the tails. This can result in paradoxical cases where increasing potential losses does not increase the estimated risk, undermining trust in the risk metric for capital allocation or regulatory purposes.

**2. Sub-additivity**

In the table, all ten risk methods satisfy this axiom. It suggests that even more complex or empirical models like bootstrapping, GARCH-based approaches, and Monte Carlo simulations maintain sub-additive behavior, thereby aligning well with real-world risk mitigation strategies. This result gives confidence that these methods are robust when applied in multi-asset or multi-factor contexts.

**3. Positive Homogeneity**

In the results, all methods satisfy this condition, including more flexible and non-parametric approaches like KDE (Kernel Density Estimation) and bootstrapping. This consistency indicates that these models behave predictably when portfolio sizes change, which is essential for tasks such as risk budgeting and leverage control. By respecting this axiom, the methods can be safely used to assess the impact of position sizing or capital reallocation without introducing scale distortion into the risk estimates.

**4. Translation Invariance**

All ten methods, including both parametric (e.g., Normal, t-distribution) and non-parametric ones, comply with this rule. This suggests that each method correctly adjusts for cash injections or hedges when calculating risk. It’s particularly relevant for risk-adjusted return metrics such as the Sharpe ratio, and for stress testing scenarios involving hedging strategies or margin requirements.

**Discussion about 5 Risk Metrics**

**1. Standard Deviation (Volatility):**

Standard deviation quantifies the total variability in returns. Among the three portfolios, the Growth portfolio shows the highest volatility (0.0189), followed by Industrial (0.0131), and then Value (0.0096). This aligns with expectations based on VaR: growth stocks are more sensitive to market changes and tend to experience larger price swings, value stocks are typically more stable and defensive. The industrial portfolio lies in between, reflecting moderate market sensitivity.

**2. Sharpe Ratio:**

The Value portfolio leads with a Sharpe of 0.0744, slightly ahead of Growth (0.0673) and Industrial (0.0643). Value achieves higher risk-adjusted returns relative to the daily risk-free rate. This indicates that although the Growth portfolio earns more in raw returns, its return per unit of risk is slightly lower, due to its elevated volatility.

**3. Sortino Ratio:**

Value (~0.1171) edges out Industrial (~0.0988) and Growth (~0.0958), indicating that Value offers a better downside-adjusted return. This further supports the defensive nature of value stocks, showing they experience fewer extreme negative returns compared to growth-oriented and industrial portfolios.

**4. Maximum Drawdown (MDD):**

Growth (-22.5%) exhibits the worst MDD, followed by Industrial (-11.6%) and Value (-12.8%). While Value had a sizable drawdown, it recovered more smoothly and consistently, unlike Growth, which likely saw a sharper crash during volatile market periods. This reveals the importance of drawdown metrics for long-term capital preservation, particularly in volatile markets.

**5. Conditional Drawdown at Risk (CDaR):**

The results show Growth again fares worst (-22.5%), with Industrial (-11.6%) and Value (-6.95%) following. Here Value’s CDaR is significantly better than its MDD, implying fewer severe drawdowns. Growth, on the other hand, experiences large and persistent losses in high-volatility periods, which is a red flag for risk-averse investors.

**Summary:**

Altogether, these metrics suggest the Value portfolio is the most risk-efficient, providing better returns per unit of downside or total risk, with smaller and less persistent drawdowns. Growth portfolios, while offering higher potential returns, come with significantly greater volatility and deeper drawdowns—highlighting their speculative nature. Industrial portfolios strike a moderate balance between the two, making them potentially appealing in diversified strategies.